

Concerning Hidden Momentum

Timothy H. Boyer

*Department of Physics, City College of the City
University of New York, New York, New York 10031*

Abstract

The fact that the author of an excellent textbook on electromagnetism could be duped by "hidden momentum" vividly illustrates the problematic nature of its use.

It greatly distresses me that the ideas of "hidden momentum," [1][2][3] which are of dubious value in the physics research literature, should now appear in discussions of dubious value or of outright error within respectable textbooks of classical electromagnetism. [4][5] It seems that the invocation of "hidden momentum" is simply a confusing distraction from the needed discussions of energy and momentum flow within systems. [6]

As an example of the outright errors caused by "hidden momentum," consider the discussion where "hidden momentum" is first invoked in an excellent undergraduate electromagnetism text. [7] In the illustrative example treated in this text, energy is transferred from a battery on the left to a resistor on the right by way of a coaxial cable. The text calculates the Poynting vector, the total electromagnetic power flow down the cable, and the total electromagnetic linear momentum located in the coaxial cable. Clearly the Poynting vector is associated with the transfer of energy and momentum from the battery (which loses energy), through the coaxial cable (which does not change), to the resistor (which gains energy). The battery clearly recoils on sending out the energy and becomes less massive, while the resistor recoils and becomes more massive on absorbing the energy. In the presentation of the textbook, it is not clear how the battery, coaxial cable, and resistor are connected together. If they are all mounted on a board with wheels, then (in response to the coupling of the recoils) we would expect the board would roll to the left so as to maintain the system center of energy at a constant location in an inertial frame where the system was initially at rest. [8]

However, rather than discussing these aspects of energy flow, the text invokes the mysteries of "hidden momentum." The text states: "In this case it turns out that there is 'hidden' mechanical momentum associated with the flow of current, and this exactly cancels the momentum in the fields." The text also makes an appeal to an irrelevant later example of "hidden momentum" where no battery and resistor are present. Thus the text makes the erroneous suggestion that the *electrical currents* in the cable carry a "hidden mechanical momentum" in the opposite direction from the electromagnetic energy flow. This is not the case.

How do we know that that the currents in the coaxial cable do not carry the mysterious "hidden mechanical momentum" invoked by the text? I believe that we can make the essential physics of the situation clearer by making the current carriers less obscure. Let's replace the current-carrying coaxial cable by two frictionless parallel filaments on which

charged beads of negligible mass are allowed to slide freely. There are positively charged beads lined up along the top filament and negatively charged beads lined up on the bottom filament, leading to an electrostatic potential between the filaments. A source of energy at the left takes a pair of beads (one positive and one negative) and moves them apart, pushing the positively charged bead onto the top filament and the negatively charged bead onto the bottom filament. On doing so, the source of energy both does work and also recoils to the left. The energy is absorbed at the right-hand end of the filaments where a positive bead is removed from the top filament while a negative bead is removed from the bottom filament, and energy is transferred as the beads are removed and brought together. The energy and momentum flow down the pair of filaments is purely electromagnetic and is given by Poynting's vector, and there is electromagnetic momentum stored in the region of the filaments. However, there is nothing even remotely resembling "hidden mechanical momentum" in the currents on the filaments. This example suggests that the comments in the text claiming the existence of "hidden mechanical momentum" associated with the flow of current are simply wrong.

Perhaps the situation is even more transparent if we consider electromagnetic energy transmitted through vacuum from the energy source on the left to an energy receiver on the right in the form of electromagnetic waves. Once again the energy source on the left will lose energy while the energy sink on the right will gain energy, and the space in between source and sink will contain both electromagnetic energy and momentum. In this case there are no electrical currents for which one can invoke "hidden mechanical momentum." Of course, in all these cases, if the energy source and energy sink are mounted on a board on wheels, the board will roll to the left due to the recoils associated with energy transmission and reception.[8]

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- [1] S. Coleman and J. H. Van Vleck, "Origin of 'hidden momentum forces' on magnets," *Phys. Rev.* **171**, 1370-1375 (1968).
 - [2] Y. Aharonov, P. Pearle, and L. Vaidman, "Comment on 'proposed Aharonov-Casher effect:

- another example of an Aharonov-Bohm effect arising from a classical lag,” Phys. Rev. **115**, 485-491 (1988).
- [3] L. Vaidman, ”Torque and force on a magnetic dipole,” Am. J. Phys. **58**,978-983 (1990).
- [4] D. J. Griffiths, *Introduction to Electrodynamics* 3rd edn (Prentice-Hall, Upper Saddle River,NJ 1999).
- [5] J. D. Jackson, *Classical Electrodynamics* 3rd edn (Wiley, New York 1999).
- [6] T. H. Boyer, ”Interaction of a point charge and a magnet: comments on ’hidden mechanical momentum due to hidden nonelectromagnetic forces,” (physics/0708.3367).
- [7] See ref. 4, pp. 356-357.
- [8] The energy flow pattern is similar to that discussed by E. F. Taylor and J. A. Wheeler in *Spacetime Physics* (W. H. Freeman, San Francisco 1963), p. 147-148.